*Network modelling*

*Network architecture*

We modelled a linear rate network of neurons obeying Dale’s law (80 % excitatory, 20 % inhibitory).  
The population dynamics are

where is a binary stimulus, is private Gaussian noise (), and the time constant is absorbed into the unit‑time scaling ().

Both the feed‑forward and recurrent weights are **rank‑one**:

, where enforcing Dale’s law.

The feed‑forward driver

ensures that each row of is non‑negative (E) or non‑positive (I).

Because is rank‑one, its spectrum contains a single non‑zero eigenvalue; the corresponding right eigenvector defines the slowest dynamical mode and, as we show below, the dominant noise direction.

*Steady‑state statistics*

For the linear system above the stimulus‑conditioned steady‑state mean and covariance are obtained in closed form:

The signal to be discriminated is the mean difference

Alignment of stimulus and noise axes

We focus on the feedback operator

whose spectral radius governs the stability of the linear‑rate dynamics. Let

be its (right) eigen‑decomposition, with eigenvectors orthonormal in the inner product. The **slow‑mode axis** is the direction that relaxes back to baseline most slowly after a small perturbation; in discrete time its relaxation constant is

so, the slowest mode is .

*Axis of correlated variability*

Private noise bypasses and therefore propagates only through the recurrent loop. Its steady‑state covariance is

Because this series weights as , variance is maximal along ; hence the **noise axis** coincides with the slow mode:

*Stimulus axis*

For a binary stimulus the mean network response is

The discriminant therefore reads

Expanding in the eigenbasis gives

Slow modes () are thus preferentially amplified.  
If, in addition, already points along (i.e.  and ) then

This tuning rule matches the adaptive‑dynamics principle from Chadwick *et al.* (2023) (ref XX): plasticity steers and so that high‑SNR feed‑forward drive excites the slowest recurrent mode.

*Consequence for our rank‑one construction*

In our model both and are rank‑one and chosen so that  
. Consequently is an eigenvector of with eigenvalue , and all of the above conditions are satisfied. Therefore

guaranteeing that the coding axis is perfectly aligned with the dominant noise direction—an essential prerequisite for the optimal‑decoding result in Figure​ 2E. Appendix B in the supplementary materials lists the general algebraic conditions for this alignment and shows that arbitrary rank‑one pairs do **not** guarantee it.

*One‑parameter family of read‑out directions*

To quantify performance as a function of mis‑alignment we define

where is any unit vector orthogonal to .

For each we compute

the signal power, noise variance, and Fisher ratio, respectively.

*Optimality of the noise axis*

Because and are even in , their Maclaurin expansions begin with quadratic and constant terms, respectively. Thus decreases faster than as soon as ; the ratio attains its global maximum at , i.e. on the noise/coding axis (Figure 2E-F).  
A full algebraic proof is given in Appendix A in the supplementary materials.

*Numerical implementation*

All quantities were evaluated analytically on a uniform grid of 181 angles ().

*Parameter summary*

| Parameter | Value | Description |
| --- | --- | --- |
|  | 120 | network size |
|  | 0.8 | excitatory fraction |
|  | 0.4 | recurrent spectral radius |
|  | 4 | feed‑forward tuning width |
|  | 1 | private‑noise s.d. |
|  | 1 | homogeneous gain |